1. $$f(x) = \frac{\sin x}{1+\cos x}$$

$$f'(x) = \frac{1}{1+\cos x}$$

$$\therefore \frac{\tilde{x}\, f'(\tilde{x})}{f(\tilde{x})} = \frac{\tilde{x}\left[\frac{1}{1+\cos\tilde{x}}\right]}{\frac{\sin\tilde{x}}{1+\cos\tilde{x}}}$$

$$= \frac{\tilde{x}}{\sin\tilde{x}}$$

(a) $\tilde{x} = 1.4$

$$\frac{\tilde{x}\, f'(\tilde{x})}{f(\tilde{x})} = 1.4207$$

$\Rightarrow$ well-conditioned

(b) $\tilde{x} = 1.0005\,\pi$

$$\frac{\tilde{x}\, f'(\tilde{x})}{f(\tilde{x})} = -2001$$

$\Rightarrow$ ill-conditioned

2. (a) When $x \neq \pi$,

$$g(x) = \frac{1+\cos x}{(x-\pi)^2}$$

$$\approx \frac{1 + \left[-1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}\right]}{(x-\pi)^2}$$

$$= \frac{1}{2} - \frac{(x-\pi)^2}{24}.$$

(b) given problem data $x = 3.16$ $\xrightarrow{\text{flt-pt.}}$ computed solution $r = 0.5908$

perturbed problem $\hat{x} = 3.16 + \varepsilon$, with $\left|\frac{\varepsilon}{3.16}\right|$ small $\xrightarrow[\text{computation}]{\text{exact}}$ $\hat{r} = \dfrac{1+\cos(3.16+\varepsilon)}{(3.16+\varepsilon-\pi)^2}$

Since $3.16+\varepsilon$ is close to π, the approximation in (a) gives

$$\hat{r} \approx \frac{1}{2} - \frac{(3.16+\varepsilon-\pi)^2}{24}$$

$$= \frac{1}{2} - \frac{(\varepsilon + 0.18407\cdots)^2}{24}$$

$$\approx \frac{1}{2} - \frac{\varepsilon^2 + .36814\,\varepsilon + .03388}{24}$$

$$\approx 0.498588 - 0.015339\,\varepsilon - \varepsilon^2/24$$

Since $\hat{r} \approx 0.498$ for <u>all</u> values of ε that are small relative to 3.16, there is <u>no</u> small value of ε for which $g(3.16+\varepsilon)$ is close to 0.5908, and thus the flt-pt. computation of $g(3.16)$ is unstable.

2. (c)

given problem,
data $x = 1.41$ $\xrightarrow{\text{flt-pt.}}$ $r = 0.3871$

perturbed problem,
data $\hat{x} = 1.41 + \varepsilon$, $\xrightarrow[\text{exact computation}]{}$ Take, for example, $\varepsilon = 0.005$
with $\left|\frac{\varepsilon}{1.41}\right|$ small

The exact value of $g(1.415)$

$$= \frac{1 + \cos(1.415)}{(1.415 - \pi)^2}$$

$$\approx 0.3874939\ldots.$$

Thus, there is a value of ε that is small relative to 1.41 for which the exact value of $g(1.41 + \varepsilon)$ is very close to 0.3871

$\Rightarrow$ the flt-pt. computation is stable.

Note: do NOT use the approximation $g(x) \approx \frac{1}{2} - \frac{(x-\pi)^2}{24}$ here to compute $g(1.415)$ because the approximation in (a) is only accurate when x is close to π. $\hat{x} = 1.41 + \varepsilon = 1.415$ is not close to π.

3. (a)

```
function root=Bisect(xl,xu,eps,imax)
i=1;
fl=f(xl);
fprintf('iteration   approximation\n')
while i<=imax
    xr=(xl+xu)/2;
    fprintf('%6.0f',i),fprintf('%18.8f\n',xr)
    fr=f(xr);
    if fr == 0 | (xu-xl)/abs(xu+xl)<eps
        root=xr;
        return
    end
    i=i+1;
    if fl*fr<0
        xu=xr;
    else
        xl=xr;
        fl=fr;
    end
end
fprintf('failed to converge in %g',imax),fprintf(' iterations\n')
```

3(b)

```
function y=f(x)
y=pi*x^2*(12.3-x)/3-45;
```

```
Bisect(0,4.1,1e-4,20);
```

iteration	approximation
1	2.05000000
2	1.02500000
3	1.53750000
4	1.79375000
5	1.92187500
6	1.98593750
7	2.01796875
8	2.03398437
9	2.04199219
10	2.04599609
11	2.04799805
12	2.04699707
13	2.04749756
14	2.04724731
15	2.04737244

```
function y=f(x)
y=(9.81*x/13.5)*(1-exp(-135/x))-40;
```

```
Bisect(1,100,1e-4,20);
```

iteration	approximation
1	50.50000000
2	75.25000000
3	62.87500000
4	56.68750000
5	59.78125000
6	61.32812500
7	62.10156250
8	62.48828125
9	62.29492188
10	62.19824219
11	62.14990234
12	62.12573242
13	62.11364746
14	62.10760498