**Principle of Mathematical Induction**

Let $P(n)$ be a proposition (true or false statement) involving an integer $n$ and suppose that we are trying to prove the following statement.

$$\forall n \geq 0, P(n)$$

Mathematical induction is the following principle.

$$\left\{\begin{array}{c} P(0) \text{ and} \\ \forall k \geq 0, \underline{P(k)} \Rightarrow P(k+1) \end{array}\right\} \Rightarrow \{\forall n \geq 0, P(n)\}$$

Strong induction is the following principle.

$$\left\{\begin{array}{c} P(0) \text{ and} \\ \forall k \geq 0, \underline{\{P(0), P(1), \ldots, P(k)\}} \Rightarrow P(k+1) \end{array}\right\} \Rightarrow \{\forall n \geq 0, P(n)\}$$

The underlined statement(s) is the *inductive assumption.* The inital $P(0)$ is the *base case*. There are lots of equivalent variants. To prove $\forall n \geq n_0, P(n)$ we can use the following form.

$$\left\{\begin{array}{c} P(n_0) \text{ and} \\ \forall k \geq 0, P(k) \Rightarrow P(k+1) \end{array}\right\} \Rightarrow \{\forall n \geq n_0, P(n)\}$$

It doesn't matter if you go from $k$ to $k+1$, or from $k-1$ to $k$. The form below and it's strong counterpart are often used for proving things about algorithms/programs.

$$\left\{\begin{array}{c} P(0) \text{ and} \\ \forall k \geq 1, P(k-1) \Rightarrow P(k) \end{array}\right\} \Rightarrow \{\forall n \geq 0, P(n)\}$$

The names $n$ and $k$ carry no special significance.

$$\left\{\begin{array}{c} P(0) \text{ and} \\ \forall n \geq 1, P(n-1) \Rightarrow P(n) \end{array}\right\} \Rightarrow \{\forall k \geq 0, P(k)\}$$

In strong induction you sometimes need more than one base case.

$$\left\{\begin{array}{c} P(0), P(1), \ldots, P(r) \text{ and} \\ \forall k \geq r, \{P(0), P(1), \ldots, P(k)\} \Rightarrow P(k+1) \end{array}\right\} \Rightarrow \{\forall n \geq 0, P(n)\}$$

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